Development of time-domain models for automotive exhaust mufflers

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The need to reduce development costs associated with exhaust system design has led to the use of new design methodologies, based on lean testing and on the use of predictive models [1]. Regarding predictive models, an essential issue is that, apart from reproducing in a suitable manner the acoustic behaviour of the muffler, they may be incorporated into a global exhaust noise prediction, so that it may be guessed in advance in the exhaust line complies with the requirements of the engine manufacturer.

Linear acoustic models are convenient for predicting the behaviour of the muffler proper; however, noise prediction tools based upon linear theory can only use a quite simplified representation of the engine as a time-invariant linear source [2], which is not the case for an engine exhaust. This inconvenience may be by-passed by means of the use of hybrid time-frequency methods, in which the engine and the exhaust manifold are described in the time domain, whilst either individual mufflers [3] or the whole exhaust line [4] are described in the frequency domain. However, these methods are quite cumbersome to use and their computational cost may be large, mostly due to the difficulty in achieving a proper convergence of the calculation.

In this sense, time-domain modelling is a convenient alternative, since any issues related with cylinder discharge and back reaction of the exhaust line on the engine are immediately taken into account. These methods make use of thermodynamic models for in-cylinder processes, which are coupled by means of suitable boundary conditions at the valve with the flow in the exhaust system, which is usually computed by means of finite differences schemes [5] or the finite volume method [6]. Obviously, the key point in such methods is the possibility to account for muffler performance, which requires the definition of suitable models representing the silencers.
In this paper, a methodology to define such models is described, and illustrated by means of its application to a particular case. First, in section 2 the basic ideas underlying the proposed models and a summary of the different steps to be considered in their definition is given. Then, in section 3 the methodology is applied to the modelling of a concentric perforated-duct muffler. Finally, the main conclusions of the work are pointed out.

Model development methodology

In the frame of time-domain simulation of exhaust systems, the assumption of one-dimensional flow is generally accepted, and forms the natural basis for the development of muffler models. However, while flow in the ducts of the exhaust system may be soundly assumed to be one-dimensional, this is not the case in general for the mufflers, which may exhibit multi-dimensional features in their frequency response. The problem is, therefore, to devise a system which is built-up with one-dimensional elements, but which may behave as a whole in a multi-dimensional manner.
In order to achieve this goal, the basic idea proposed here is to actually build an equivalent muffler model, whose geometry need not be the same as that of the real muffler, but might be a different one, with the only requirement that the same attenuation mechanisms as in the real case are present. This implies that the main geometric characteristics defining the acoustic response of the muffler should be identified, either from experiment or from complex calculations. Obviously, the initial requirement that the basic elements considered in the model are themselves one-dimensional (or even zero-dimensional, as in the case of ideal cavities) will establish a certain upper frequency limit for the validity of the model so conceived. This frequency limit should be consistent with the relevant engine orders as specified by the engine manufacturer.

The definition of such a model with reasonable expectations of success may be achieved by using the methodology summarised in the following steps:

Identification of the main geometric characteristics influencing the acoustic behaviour of the muffler, most notably, longitudinal or transversal propagation directions and their associated lengths. Ideally, this information should be obtained from experimental parametric studies performed in controlled conditions and sufficiently representative of the real exhaust working conditions. Probably, the best solution in this sense is given by the use of an improved impulse technique [7] that, moreover, allows for direct time-domain simulation.

Identification of the main attenuation mechanisms acting inside the muffler. This information is best obtained from the use of complex multi-dimensional calculations, when available for the case considered. While this step may provide additional insight into the basic behaviour of the muffler, it may not be necessary if the interpretation of the results obtained in step 1 is sufficiently clear.

Development of an equivalent muffler model, this is, a physical system whose acoustic behaviour, for the frequency range specified, is the same as that of the real muffler. It should be stressed that, at this point, the model is not related to any particular set of equations, save for the general assumption of one-dimensional propagation through its basic elements.

Implementation of the model into some computational tool, so that its performance may be verified.

Preliminary validation and eventual refining of the model. In this step, a suitable reproduction of the experimental results obtained in step 1 is the crucial point, since this allows a first assessment of the validity of the model, as well as checking the upper frequency limit. Moreover, eventual sources of dissipation may be identified, which may lead to the inclusion of resistive terms in the model.

Validation in engine. This last step is essential in order to establish the ability of the model developed to reproduce the behaviour of the muffler in the real exhaust, from both the point of view of noise attenuation and back-reaction on the engine.

As can be seen, this methodology reduces to a minimum experimental work on real engine, putting the stress on the impulsive measurements. This has two main consequences: on one hand, a representative model may be obtained at a relatively low cost; on the other hand, it is imperative that the results obtained from the impulsive measurements are sufficiently realistic, so that they may soundly be extrapolated to engine conditions.

Application to a concentric perforated-duct muffler

As an example, the methodology described in section 2 will now be applied to the case of the concentric perforated-duct muffler, one of the simplest and most widely used schemes for front mufflers [8]. A scheme of this muffler together with the notation used is shown in figure 1.

The classical linear approach for the model of these mufflers is the segmentation approach [9] which leads to an extremely involved mathematical representation which, in the end, is extremely sensitive to the impedance attributed to the perforated section. Actually, the flow at the perforations is extremely unstable, which can lead to non-linear issues for

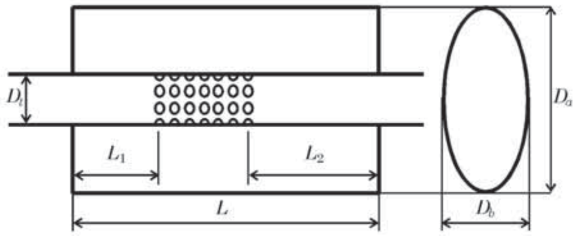

Fig. 1 : Scheme and notation for perforated-duct muffler

relatively high excitations [10]. This segmentation approach, however, forms the basis of any reasonable approach to the modelling of this muffler, mostly when models are to be implemented in time-domain calculations [11,12,13].

In the following paragraphs the different steps in the development of the model will be described.

Experimental study

In view of the muffler structure shown in figure 1, an experimental study covering reasonable ranges of all the relevant parameters was performed, so that all the relevant effects as well as their eventual interactions could be ascertained. The following parameters were varied:

- The shape of the chamber cross-section (circular or oval, with different eccentricities)
- The total length of the chamber, L.

The lengths of the inlet and outlet extended ducts, L_1 and L_2.

The fraction of perforated area in the main duct, defined as:

$$\frac{A_p}{A_t} = \frac{ND_h^2}{D_t^2},$$

where A_p is the perforated area, A_t is the duct area, N is the number of holes and D_h is their diameter.

- The perforation pattern (staggered or in-line arrangement, diameter of the holes and distance between them).

From all the measurements performed, two comparisons will be discussed here that evidence the main features in the acoustic behaviour of the muffler. In figure 2, two configurations differing only on the position of the perforated section (that is, in the length of the extended ducts) are compared. This comparison shows that pass-bands associated with the total length of the chamber appear, regardless of the

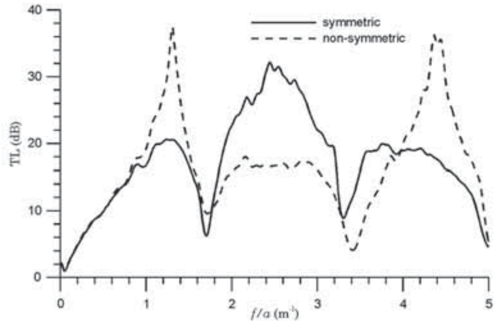

Fig. 2 : Influence of extended ducts length

system. Moreover, the existence of “quarter-wave” effects is evidenced, both in the symmetric ant the non-symmetric case, obviously at different frequencies related with the length of the extended ducts.

A further ingredient may be guessed when analysing figure 3, where two configurations differing only in the perforated area ratio are compared.

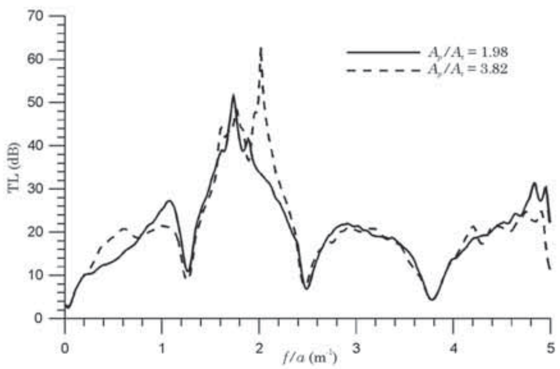

Fig. 3: Influence of the perforated area ratio

In this case, any issues related with quarter-wave effects should approximately be the same. However, a duplication of the main attenuation spike is observed, with a new characteristic frequency being evident. This effect is related with the lateral resonance of the system as a whole [14], which can be qualitatively justified as follows. This effect should be, at least for sufficiently low frequencies, similar to that associated with an equivalent Helmholtz resonator, whose characteristic resonance frequency is given by :

$$\frac{f}{a} = \frac{1}{2\pi}\sqrt{\frac{A_n}{L_n V}}$$

where a is the speed of sound, A_n and L_n are, respectively, the cross-section and the length of the resonator neck, and V is the resonator volume. In the comparison shown in figure 3, the shift to higher frequencies is consistent (even quantitatively) with the increase in the equivalent area of the neck.

As a summary of the experimental study, the three main ingredients to be incorporated into the model are :

- The total length of the chamber (main pass-bands),
- The quarter-wave effects related with extended ducts,
- The coupling of the main duct and the chamber in a whole resonant system.

Model development

According to the previous points, and considering the usual approach to these silencers, the general scheme of a possible model is shown in figure 4.

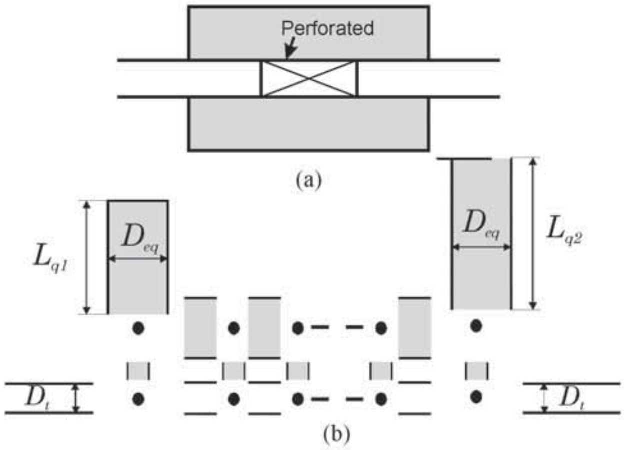

Fig. 4: Initial model

The effect of the extended ducts has been represented by means of quarter-wave resonators with an equivalent diameter given by :

$$D_{eq}^2 = D_a D_b - D_t^2$$

and with equivalent lengths given by :

$$L_{q1} = L_1 + \delta \quad ; \quad L_{q2} = L_2 + \delta$$

where δ is a length correction related with the porosity [15]. In principle, each row of holes would be represented by an equivalent duct connecting the main duct and the chamber, according to the usual segmentation approach. However, it will be shown next that, depending on the upper frequency limit established, this model admits further simplification.

In figure 5 the influence of the number of connections is studied for frequencies below 5a at usual room temperature (more that sufficient for practical purposes).

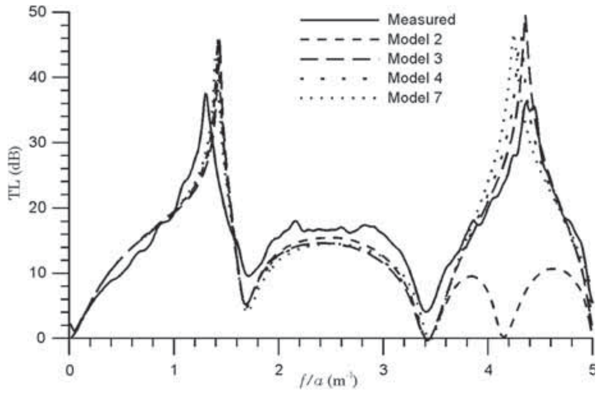

Fig. 5: Influence of the number of connections

It can be observed that with a single intermediate connection an acceptable reproduction of the frequency response is obtained in the frequency range considered. In passing, it can be observed that even in the absence of any connection (curve labelled model 2) an excellent agreement is obtained for frequencies below 3a. Therefore, the simplified model shown in figure 6 will be considered.

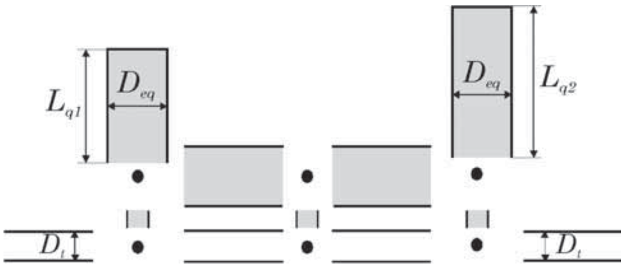

Fig. 6: Simplified model

According to the methodology proposed, results computed with this simplified model will now be compared with impulsive measurements, through the time-domain simulation of the experimental set-up. As an example of this comparison, in figure 7 results for two different symmetric configurations are shown, with different values of the perforated area ratio. It can be observed that all the main issues in the experimental curve have been reproduced, although the model underestimates the dissipation at the attenuation spikes. Actually, this effect is more clear in the case with a high perforated area ratio. This effect may be taken into account by a proper adjustment of the friction coefficient in the ducts simulating the perforations, so that any non-linearity associated with the flow at the holes may be approximately simulated.

Moving to non-symmetric configurations, a similar comparison is shown in figure 8. The conclusions now are almost the same: the main features of the muffler have been reproduced, but some improvements in the details of the perforations are possible, mostly in what is referred to dissipation at the spikes and the pass-bands.

In any case, the model seems reasonably accurate and will thus be checked against real engine measurements.

Validation in engine conditions

Final assessment of the model was obtained from measurements performed in real engine conditions. For this purpose, an exhaust line comprising a catalytic converter, a concentric perforated-duct muffler (as the from muffler) and a simple expansion chamber (as the rear muffler) was considered, in order to minimise uncertainties in the evaluation of the front muffler model. A spark-ignition two-litre engine was used.

The experimental facility is shown schematically in figure 9: the test cell is located adjacent to a semi-anechoic chamber where the exhaust tailpipe is mounted flush to the reflecting wall, allowing for the measurement of exhaust noise in free-field conditions. Moreover, instantaneous pressure at selected points in the exhaust line was recorded, in order to dispose of additional criteria for model evaluation.

The model structure devised was input to a conventional time-domain calculation now incorporating a suitable representation of the engine, and thus allowing for computation of instantaneous pressures as well as the estimation of exhaust noise (by means of a simple monopole emission model).

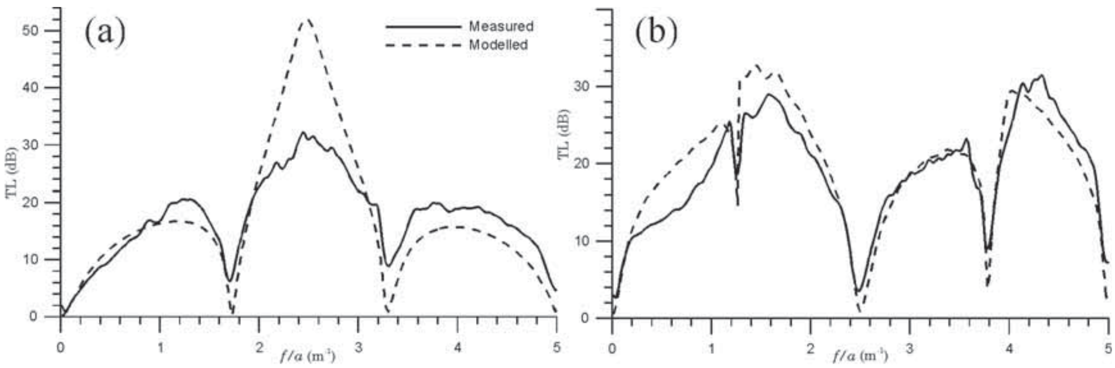

Fig. 7: Symmetric cases: (a) high area ratio; (b) low area ratio.

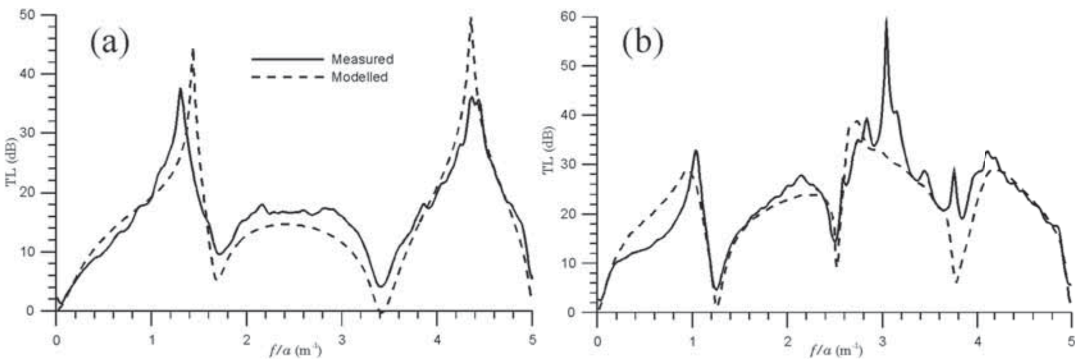

Fig. 8: Non-symmetric cases: (a) high area ratio; (b) low area ratio.

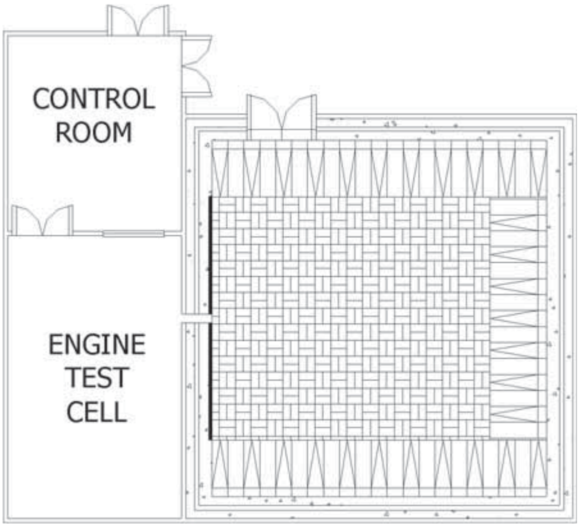

Fig. 9: Experimental set-up: semi-anechoic chamber

As an example of the results obtained, in figure 10 comparison between measured and modelled instantaneous pressures upstream and downstream of the concentric resonator are shown at an engine speed of 1250 rpm. These results indicate that, at least for the low frequencies, the model is able to reproduce the behaviour of the muffler in realistic engine conditions. This is particularly important from the point of view of the influence of the exhaust line on engine performance, which can only be properly described in the time domain.

More information on the model performance may be obtained looking at the frequency domain representation given in figure 11, where the same points and conditions as in figure 10 are considered. It may be observed that the agreement obtained is good for a sufficient number of engine orders, which indicates the practical applicability of the model proposed.
Another interesting magnitude from the point of view of noise prediction is the pressure at the tail-pipe. Additionally, the accurate prediction of tail-pipe pressure is a good index of the quality of the modelling, since the proximity of the open end makes the results of the calculation extremely sensitive to errors in temperature estimation, due to the superposition between the pressure wave coming from the engine and that reflected by the open end.

Comparison between measured and modelled tail-pipe pressures is given, in the time domain, in figure 12, for two different engine speeds. Bearing in mind the above comments on the sensitivity of this magnitude, the results of the modelling may be regarded as mostly satisfactory, at least for the low frequencies.

The same comments apply when considering the corresponding spectra, shown in figure 13: an acceptable representation has been obtained for the first engine orders, thus indicating that a reasonable representation of the flow at the tail-pipe has been achieved. In some sense, this provides a direct validation of the calculation performed, without the unavoidable uncertainties in exhaust noise prediction, related with the eventual accuracy of the emission model.

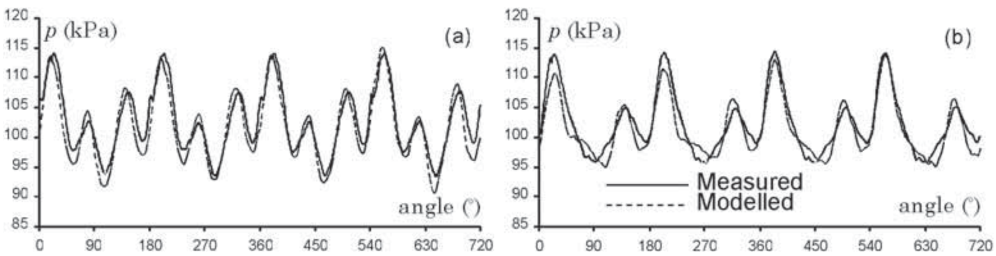

Fig. 10: Instantaneous pressure: (a) upstream; (b) downstream

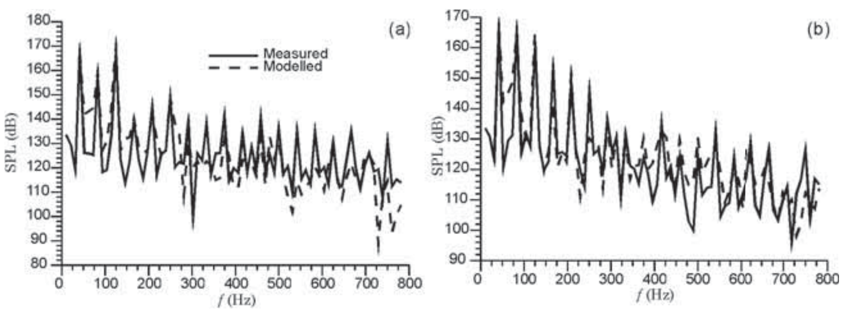

Fig. 11: Pressure spectra: (a) upstream; (b) downstream

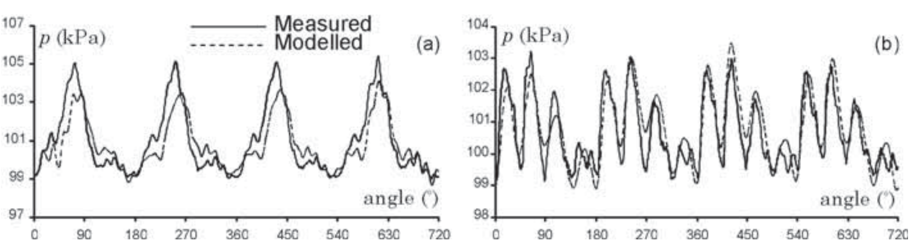

Fig. 12: Tail-pipe pressure: (a) 1250 rpm; () 4250 rpm.

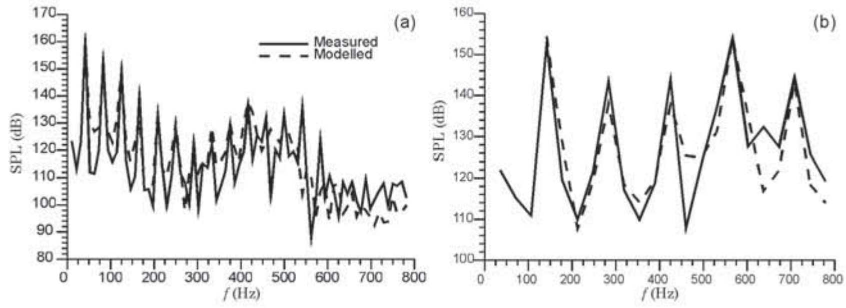

Fig. 13: Tail-pipe pressure spectra: (a) 1250 rpm; (b) 4250 rpm.

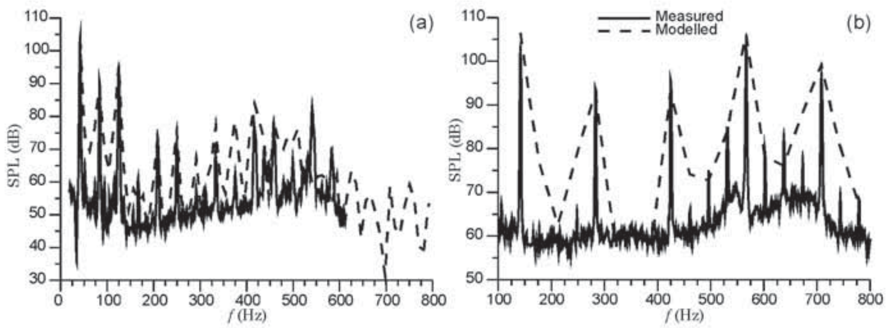

Fig. 14: Exhaust noise spectra: (a) 1250 rpm; (b) 4250 rpm.

Finally, comparison between measured and modelled exhaust noise spectra is given in figure 14, for the same two engine speeds considered previously.

Again, the results indicate the accuracy of the model proposed, giving a sufficient assessment of the quality of the whole modelling, including, of course, the model developed for the perforated-duct muffler considered.

Summary and conclusions

A general methodology for the development of muffler models to be used in time-domain exhaust noise calculations has been presented. The methodology is based on:

Measurements performed in controlled but realistic conditions which allow for the identification of the main features influencing the acoustic behaviour of the muffler and for the preliminary assessment or improvement of the model.

The representation of the muffler by an equivalent acoustic system, composed of simple zero- and one-dimensional elements, which incorporate the main attenuation mechanisms present in the muffler.

The application of this methodology has been shown in the case of a concentric perforated-duct muffler. A simple model has been devised that, when incorporated into a global exhaust noise prediction code, has yielded satisfactory results, both for in-duct magnitudes as instantaneous pressure and for exhaust noise.

Acknowledgements

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