Sources of uncertainties in vibro-acoustic simulations

The paper addresses the main sources of uncertainties in vibro-acoustic simulations. Such simulations involve the selection of a particular continuous model already characterized by some simplifications of the real physical behavior. Material data, geometrical parameters and boundary conditions (as, for example, excitations) are additional sources of uncertainty. Last but not least, the discretization process supporting the numerical model introduces additional errors related to computational algorithms (interpolation, integration, equation solving). The paper reviews these uncertainties for numerical methods (FEM, SEA) traditionally used for vibro-acoustic simulations. Some trends for securing vibro-acoustic simulations in such a context are also presented.

Cet article présente les sources principales d'incertitudes dans les simulations vibro-acoustiques. De telles simulations impliquent la sélection d'un modèle continu particulier déjà caractérisé par quelques simplifications du comportement physique réel. Les données matérielles, les paramètres géométriques et les conditions aux limites (comme, par exemple, des excitations) sont des sources supplémentaires d'incertitudes. Dernière et non des moindres, la méthode de discrétisation qui sous-tend le modèle numérique introduit des erreurs supplémentaires relatives aux algorithmes informatiques (interpolation, intégration, résolution d'équations).
L'article passe en revue ces incertitudes pour des modèles numériques (FEM, SEA) traditionnellement utilisés pour des simulations en vibro-acoustique. Quelques orientations pour sécuriser les simulations vibro-acoustiques dans un tel contexte sont aussi présentées.

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In the last years, numerical simulations have been more and more intensively used in industrial design processes to assess the vibro-acoustic behavior of various components. The problem of constructing an appropriate numerical model from a real physical configuration is however complex and directly involves several sources of uncertainties. In this context, non-deterministic methods can be defined as any methods aimed at qualifying or quantifying the uncertainty on response or performance indicators supported by a specific problem.

The uncertainty sources can be classified in the three following categories [1,2] :

- First, the lack of knowledge in the physical process or the complexity of this process forces the designer to define a set of modeling assumptions during the construction of the mathematical model. Modeling errors result from these assumptions. Practice and intuition of the designer as well as sophistication of the model generally tend to reduce the modeling error.
- The second source of uncertainty is linked to the numerical errors involved by the computational implementation of the mathematical model. The numerical methods (interpolation, integration, equation solving, etc.) are generally selected on the base of their convergence properties (which should lead to a reduction of the numerical error as soon as the size of the numerical model is increased). This refinement however results in higher computational requirements. Depending on the numerical method, some procedures are available to estimate this numerical error. It can thus be said that, basically, this second source of uncertainty is linked to the efficiency of the available computing tools.
- The third source that involves uncertainty is related to the totally unknown or intrinsically variable parameters of the mathematical model as well as to their experimental identification. The resort to laboratory or experimental procedures suggests that the exact knowledge of the parameters is not possible. In fact, these procedures always need an answer to representativeness issues, reproducibility features and measurement errors. In some circumstances (especially in the early design phase), only rough descriptions (in terms of intervals or linguistic values ('low', 'high')) of material and/or geometrical parameters are available. A finer classification of model parameter uncertainties is often made: uncertainty on loads or excitations, uncertainties on strength properties and uncertainties on material, mechanical and geometrical properties of the model.

This convenient description should not be taken as sufficient as these three uncertainty sources are not independent. It is, for instance, difficult to reduce the first uncertainty source without increasing the last one, as a complex model very often relies on an important number of parameters. A good example is provided by porous materials [3] : the most simple model relies on a concept of equivalent fluid (assuming a rigid skeleton) while more sophisticated models are relying

on a two-phase model (Biot model for instance) involving a substantial number of parameters not easy to measure and quite frequency dependent …

In this context, the steady development of computational mechanics over the past five decades, supported by continuously more efficient computing tools, gives the opportunity to define structural models reaching a relatively high level of refinement. These models however require the definition of many material properties, geometrical parameters, load conditions, ..., the exact knowledge of which could not be possible. Some compromise has therefore to be found.

In a vibro-acoustic context, the general modeling strategy should take into account the fact that uncertainties are usually higher for the (structural) dynamic component than for the acoustic component. This is due to the relative simplicity of the acoustic propagation operator with respect to the elastodynamic operator. In most of the cases, the quality of acoustic predictions strongly depends on appropriate boundary conditions (especially velocity or acceleration constraints).

An important source of uncertainty is attached to boundary conditions (in general) and, particularly, to loading or kinematic excitations of the model. Here, uncertainties occur simultaneously on the mathematical model stage (nature of the excitation, spatial distribution, spectral content or time histories, …), on the numerical model stage (discretization of the selected excitations, sampling procedures, …) and on the parameter definition stage (intensities, probability distributions of the loads, stationarity or unstationarity of the random processes…). Methods for handling uncertainties in the load conditions of dynamic models are now well established.

The related semi-empirical spectral models for usual random distributed loading (diffuse field, turbulent boundary layer) are important components of vibro-acoustic simulations. This is, at least partially, due to the difficulty to simulate highly turbulent unstationary flows with traditional CFD tools at high Reynolds number.

The uncertainty sources described above are also hard to quantify independently. More precisely, it is difficult to determine the uncertainty level implied in the modeling stage, that is involved during the mathematical modeling process and its numerical implementation. Practically, there are no other means than to consider that these uncertainty sources are negligible or are covered by the variability of the model parameters. This last assumption, although it has no physical foundation, seems to be generally accepted in the literature. Recently Soize [4] developed non-parametric approaches in order to handle these uncertainties in a more rational way. This model has been extended recently to vibro-acoustic studies [5].

Based on the above observations, one is forced to admit that a given amount of uncertainty or irreducible variability is present in each vibro-acoustic model, and that simply carrying out a deterministic analysis leads to an error, which should be at least estimated. Non-deterministic approaches are thus a natural and necessary extension of present analysis techniques.

Mechanical and geometric uncertainties

Stochastic Finite Elements

Amongst all numerical procedures in non-deterministic computational mechanics, the stochastic finite element method (SFEM) [6,7] has been developed and applied to the reliability and response variability assessment of static and dynamic, linear and non-linear problems. In the context of second moment approaches for the response variability assessment, the perturbation SFEM [8], the spectral SFEM [9] and the Monte-Carlo simulation method are available.

The selection of a particular method often relies on considerations about computational requirements with regard to the dimension of the problem (number of degrees of freedom (dofs) and number of uncertain parameters) and to the considered variability level. The Monte-Carlo simulation, from its general formulation, is able to cope with high variability levels but suffers from prohibitive computational costs. Recent developments have thus been aimed at optimizing its application by using variance reduction techniques or by resorting to parallel computing [10]. In the spectral SFEM, the number of involved random variables is very critical so that appropriate algebraic solvers, based on the particular structure of the problem, have been developed [11]. The perturbation method has been widely applied to stochastic problems since it usually requires low computational resources. It however suffers from the fact that it relies on a low-degree polynomial approximation of the structural response and is thus aimed at solving models involving a low variability level of the design parameters [2,12].

Considering a linear vibro-acoustic problem stated in the frequency domain, the discretization using conventional finite elements leads to the following complex-valued algebraic system:

$$\begin{pmatrix} -\omega^2 M_s + i\omega C_s + K_s & i\omega C_{FS} \\ i\omega C_{FS}^T & -\omega^2 M_F + i\omega C_F + K_F \end{pmatrix} \cdot \begin{pmatrix} U_S \\ \Psi \end{pmatrix} = \begin{pmatrix} F_S \\ F_F \end{pmatrix} \quad (1)$$

where ω is the circular frequency; $\mathbf{U_S}$ is the vector of nodal displacements of the structure; Ψ is the vector of nodal potentials of the fluid; $\mathbf{F_I}$ is the excitation vector, $\mathbf{M_I}$, $\mathbf{C_I}$ and $\mathbf{K_I}$ are the mass, damping and stiffness matrices for the fluid (I=F) or the structure (I=S) and $\mathbf{C_{FS}}$ is the fluid-structure coupling matrix. Defining a set of random variables (either of material or geometrical nature) $b_i(\theta)$ (i=1, …, q), each of the above vectors and matrices is a random quantity and implicitly exhibits a θ-dependence. Note that this system may be rewritten in a canonic form for dynamical systems:

$$\left(-\omega^2 \mathbf{M}(\theta) + i\omega \mathbf{C}(\theta) + \mathbf{K}(\theta)\right) \cdot \mathbf{U}(\theta) = \mathbf{F}(\theta), \quad (2)$$

where $\mathbf{U}(\theta)$ is the vector of both potentials or displacements, $\mathbf{F}(\theta)$ is the excitation vector, $\mathbf{M}(\theta)$ is the system mass matrix, $\mathbf{C}(\theta)$ is the system damping matrix and $\mathbf{K}(\theta)$ is the system stiffness matrix.

Monte-Carlo simulation

Solving this system of equations for a relatively high number of realizations of the random design variables leads to a sampling of the nodal values of the unknown potentials and displacements. Post-processing operations can then be applied to these basic values in order to get samples of local response indicators (such as the pressure field) or of global indicators (such as the mean quadratic pressure over the domain). Statistical convergence properties guarantee that applying statistical operators to the produced samples gives estimates of the parameters of the response distribution laws. It is however difficult, for industrial problems, to obtain (even low order) converged statistics due to the high number of repeated simulations. In addition to poor convergence properties, Monte-Carlo simulation suffers from the following drawbacks:

- in the context of high variability level (where alternative non-deterministic methods are not efficient), a significant dependence of the response distribution law on the parameter distribution law is possible;
- no error estimates are available to investigate the accuracy of the computed response statistics.

Sensitivity analysis and perturbation approach in physical coordinates

The perturbation SFEM basically assumes that the random vibro-acoustic response can be represented by a first-order Taylor development in the neighbourhood of the mean configuration:

$$U(\theta) = \overline{U} + \sum_{i=1}^{q} \frac{\partial U}{\partial b}\left(b_i(\theta) - \overline{b_i}\right), \qquad (3)$$

By first-order differentiating Eq. (2) with respect to a given parameter b, the following relation is obtained:

$$\left(-\omega^2 M + i\omega C + K\right)\cdot\frac{\partial U}{\partial b} = \frac{\partial F}{\partial b} - \left(-\omega^2\frac{\partial M}{\partial b} + i\omega\frac{\partial C}{\partial b} + \frac{\partial K}{\partial b}\right)\cdot U \qquad (4)$$

with the quantities **K**, **C**, **M**, **U** and **F** being evaluated at the nominal value of b. This relation shows that the first-order sensitivity of the unknown potentials and displacements can also be evaluated by solving a complex-valued algebraic system. It is noticeable that this system has the same operator as the zeroth-order system, which drastically reduces the computational effort required for the solution (as the system already exists in factorised form). The right-hand side of the first-order algebraic system involves the sensitivities of the **K**, **C**, **M** matrices and of the excitation **F** with respect to the design parameter b.

Applying first-order and second-order statistical operators to the approached response surface directly leads the mean and variance of the response indicators. Statistics of post-processing fields can be obtained similarly. It is clear that the global accuracy achieved by the perturbation SFEM depends on 1) the variability level of the design variables and 2) the nonlinearity degree of the implicit relation between the response indicator and the design variables [2].

Sensitivity analysis and perturbation approach in modal coordinates

The solution strategy in physical coordinates (direct approach) described above should be distinguished from a modal approach in which the eigenmodes of the system (and their associated sensitivities) are first identified and used as a base for developing the unknown response (and its associated sensitivities). Referring to the previously-stated linear vibro-acoustic problem in the frequency domain, the structural and acoustic eigenmodes, (ω_{Si},Φ_{Si}) (i=1,…,n_S) and (ω_{Fj},Φ_{Fj}) (j=1,…,n_F) resp., are extracted prior to the frequency analysis. The unknown potentials and displacements can then be projected on their respective modal bases:

$$\begin{cases} U_S = \sum_{i=1}^{n_S} \alpha_{Si}(\omega)\Phi_{Si} \\ \Psi = \sum_{j=1}^{n_F} \alpha_{Fj}(\omega)\Phi_{Fj} \end{cases} \qquad (5)$$

and the modal coordinates are obtained at each discrete frequency by solving the following system of equations of order n_S+n_F:

$$\begin{pmatrix} \Phi_S^T\cdot Z_s\cdot\Phi_{Si} & i\omega\Phi_S^T\cdot C_{FS}\cdot\Phi_F \\ i\omega\Phi_F^T\cdot C_{FS}^T\cdot\Phi_S & \Phi_F^T\cdot Z_F\cdot\Phi_F \end{pmatrix}\cdot\begin{pmatrix}\alpha_s\\ \alpha_F\end{pmatrix} = \begin{pmatrix}\Phi_S^T\cdot F_S\\ \Phi_F^T\cdot F_F\end{pmatrix} \qquad (6)$$

Observe that, due to the convergence properties of the modal superposition, the low-frequency range analysis does not require a full extraction of the eigenmodes of the structure and the fluid domain. A reduction of the problem dimension is consequently achieved in this way.
For first-order sensitivity analysis, the modal superposition is first-order differentiated, which leads to the following relations:

$$\begin{cases} U_s = \sum_{i=1}^{n_S}\left(\frac{\partial\alpha_{Si}(\omega)}{\partial b}\Phi_{Si} + \alpha_{Si}(\omega)\frac{\partial\Phi_{Si}}{\partial b}\right) \\ \Psi = \sum_{j=1}^{nF}\left(\frac{\partial\alpha_{Fj}(\omega)}{\partial b}\Phi_{Fj} + \alpha_{Fj}(\omega)\frac{\partial\Phi_{Si}}{\partial b}\right) \end{cases}. \qquad (7)$$

The sensitivities of the modal coordinates are obtained by solving the complex-valued system of equations:

$$\begin{pmatrix} \Phi_S^T \cdot Z_S \cdot \Phi_s & i\omega \Phi_S^T \cdot C_{FS} \cdot \Phi_F \\ i\omega \Phi_F^T \cdot C_{FS}^T \cdot \Phi_s & \Phi_F^T \cdot Z_F \cdot \Phi_F \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial \alpha_s}{\partial b} \\ \frac{\partial \alpha_F}{\partial b} \end{pmatrix} = \begin{pmatrix} \frac{\partial \Phi_s^T}{\partial b} \cdot F_s \\ \frac{\partial \Phi_F^T}{\partial b} \cdot F_F \end{pmatrix} + \begin{pmatrix} \Phi_S^T \cdot \frac{\partial F_s}{\partial b} \\ \Phi_F^T \cdot \frac{\partial F_F}{\partial b} \end{pmatrix} + \begin{pmatrix} \frac{\partial}{\partial b}\left(\Phi_S^T \cdot Z_s . \Phi_s\right) & i\omega \frac{\partial}{\partial b}\left(\Phi_S^T \cdot C_{FS}^T . \Phi_F\right) \\ i\omega \frac{\partial}{\partial b}\left(\Phi_F^T \cdot C_{FS}^T . \Phi_S\right) & \frac{\partial}{\partial b}\left(\Phi_F^T \cdot Z_F . \Phi_F\right) \end{pmatrix} \cdot \begin{pmatrix} \alpha_s \\ \alpha F \end{pmatrix} \quad (8)$$

Note the close similarity between these relations and the sensitivity analysis in physical coordinates. Once the system is projected in the modal basis, the sensitivity analysis can be performed using similar solution sequences, which can lead to a substantial profit in an algorithmic implementation.

An alternative to solving the above equation for the sensitivities of the modal coordinates is to perform Monte-Carlo simulation on the equation giving the modal coordinates [12]. In fact, this discrete system is of reduced size due to the modal formulation and repeated solutions are not anymore computationally-intensive with regard to the initial eigenproblem solution. Moreover, using Monte-Carlo simulation enables to handle the nonlinearity between the modal coordinates and the random design parameters without any restriction on the variability level (which is especially useful at resonances).

Random field modeling

The concept of random field [13] is often resorted to as a mean to model the spatial variability of the material parameters (sometimes improperly considering the lack of experimental knowledge in the uncertain spatial behaviour of material properties). As a consequence of its continuous character, the random field requires an appropriate discretisation, leading to the identification of a finite set of random variables. This set has to satisfy two opposite requirements: on the one hand, it should represent accurately the original continuous random field and, on the other hand, it should involve the smallest number of random variables since the computational cost of response variability analysis grows significantly with this number. For instance, the spectral SFEM, based on the Karhunen-Loeve expansion of the random field, uses polynomial chaoses on which the stochastic response is projected [9,11].

The identification of this projection requires the solution of an algebraic system of order P x N, where N is the size of the deterministic problem and P is the number of terms (basic random polynomials) involved in the projection. This number P grows prohibitively as soon as the global order of the method and/or the number of discrete random variables are increased. The Monte-Carlo simulation method is known to provide the best variability estimations as soon as the number of samples involved in the analysis is sufficient. Whereas it is difficult to estimate this number, it should be increased with the number of random variables involved in the model, which results in a substantial increase of the computational time. Finally, the perturbation method, based on a low-order response representation, estimates the response variability at a relatively low additional cost with respect to the deterministic analysis, which is however proportional to the number of random variables in the stochastic analysis.

Reduction techniques of the finite set of random variables are however available [2]. The first well-known reduction technique relies on the finite element mesh to perform the discretization of the random field. A midpoint technique enables the identification of a set of correlated random variables from the covariance function of the random field, each variable being related to a particular element. A decorrelation procedure based on the spectral analysis of the discrete covariance matrix is performed in order to identify a reduced number of stochastic basic components. As an alternative to the midpoint discretization, a (numerical) Karhunen-Loeve decomposition is possible, the truncation of which allows to extract the stochastic components that introduce the major variability in the model.

The criteria leading to the definition of the random field discretization and analysis meshes are different, the first being related to the correlation length of the random field, the second being related to the stress gradients or the wave speeds in the model. Generally, the second criteria is more demanding than the first one. It results from this observation that using the finite element mesh for the random field discretization can lead to increase computational costs and even to numerical inaccuracies. A second reduction technique is thus possible in which a different discretization mesh is used.

In contrast with random variable models, the random field model enables the development of a compensation effect, i.e. a reduction of the response variability due to the correlation structure of the random field. Fig. 1 illustrates, for the simple configuration of a clamped-free beam with random flexibility F (analytical solution is available [2]), the close relation between the compensation effect and the filtering of the stochastic components of the random field by the dynamic system. First, the intensity of the compensation effect is related to the correlation structure of the random field. A more important compensation effect is observed in the case of a low correlation length.

The error that would be obtained if the correlation properties of the field was wrongly taken into account would thus be notable. Second, the compensation effect strongly depends

on the localization of the response observation point (the larger the development area of the compensation, the larger the compensation effect). For global response indicators, the compensation intensity is thus important.

Practically, the selection of the random field discretization mesh and the truncation ratio should be based on the following considerations.
First, depending on the random field type and correlation structure, it is possible to evaluate the theoretical number of

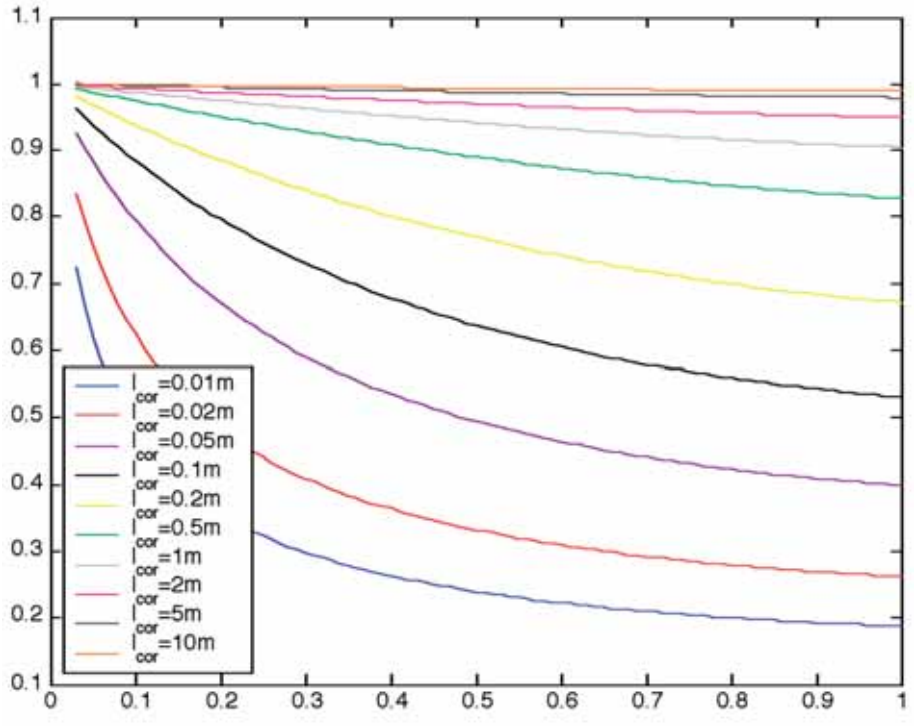

Fig. 1: Compensation effect on the displacement field u of a

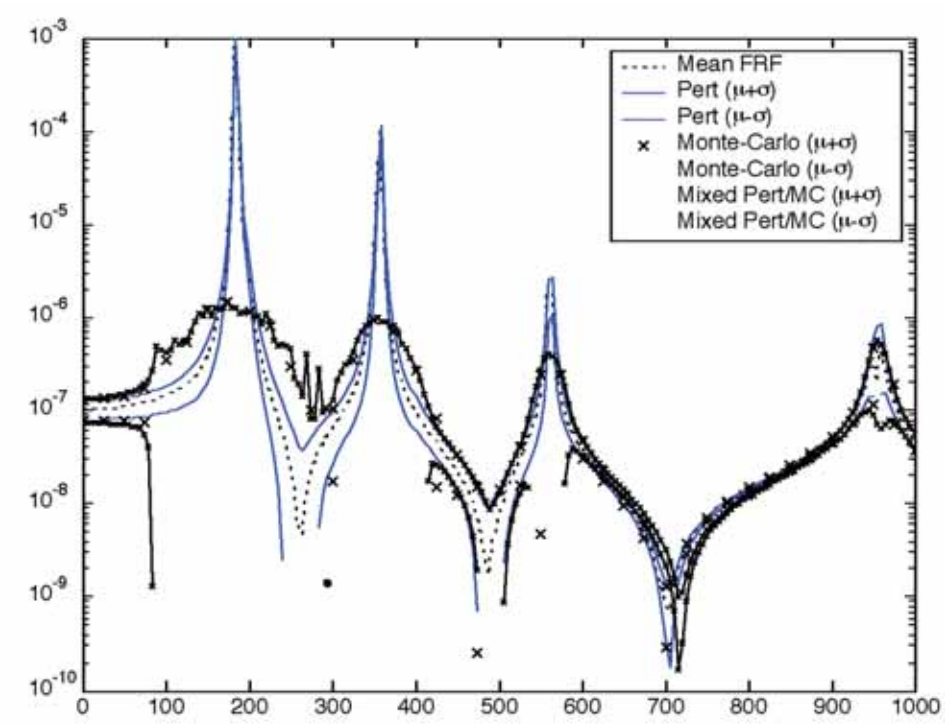

Fig. 2 : Dispersion of the vertical displacement FRF for a plate with random flatness default

random variables NKV giving a good variance representation of the field.
Secondly, considering the possibility of the development of a compensation effect (depending on response characteristics and random field parameters), this theoretical number can be reduced.
Thirdly, the discretisation mesh is selected in such a way that it is compatible with the discretisation technique (a finer mesh should be selected for the midpoint discretisation than for the Karhunen-Loeve expansion technique). This mesh should allow for a good representation of the random field (sufficient number of subdivisions per correlation length and per eigenfunction wavelength). If we note by NEL or NND the number of elements and nodes of this mesh, the truncation ratio r is finally selected in such a way that r x NND=NKV or r x NEL=NKV for the Karhunen-Loeve expansion or midpoint discretisation, respectively.

Shape uncertainty

Most SFEM applications in the literature involve uncertain parameters of material or mechanical nature and rarely of geometrical nature. Random shape variables result in uncertain domains and boundaries, which complicate the stochastic analysis. Generally, applications aimed at characterizing the response variability resulting from a geometric uncertainty source are based on models allowing particular kinematic assumptions. In such cases, the treatment of the geometric variables by the same means as the material parameters (for instance the cross-section of a beam element or the thickness of a plane stress model) is possible. Only a few applications really handle the shape randomness. However, geometric uncertainties can be found in a wide range of mechanical applications and should consequently be addressed in a general framework.

The perturbation SFEM, used in conjunction with the shape design sensitivity theory, offers a possibility to develop an efficient technique for handling geometric uncertainties [12].

As an example, Fig. 2 shows the variability of the vertical displacement field for a plate having a random flatness default. Due to the apparition of stiffening membrane effects, the displacement field exhibits an hypersensitive behavior w.r.t. the low non-planearities [14].
This effect should be accounted for in explaining the scatter in the vibro-acoustic behavior of industrially identical structures (see Fig. 2).

Fuzzy Finite Elements

Fuzzy logic and fuzzy arithmetics have also been applied to the field of finite element analysis. In fact, not all uncertainties are objectively quantifiable, especially those based on incomplete information, and can therefore not be handled satisfactorily in the probability theory. In fuzzy FE analysis, each uncertain property and each response value is represented by a fuzzy number, defined by its membership function. Fuzzy numbers are then discretized by performing cuts at given degrees of membership, which reduces the analysis to the solution of interval arithmetic problems.

The use of interval arithmetics however restricts the tractability of the method to small systems (the problem inversion being computationally tedious) and, if not carefully controlled, leads to an artificial augmentation of the interval sizes. Circumventing this spurious behavior, the combinatorial approach (vertex method), consists in considering all possible combinations for the upper and lower bounds of the design parameter intervals. Specific modal-based methods for the structural dynamics in the frequency domain have been proposed by Moens and Vandepitte [15].

Recent results [16] tend to show that adding sensitivity-based information in the construction of the fuzzy response enables to achieve better efficiency in the numerical procedure. As an example, Fig. 3 shows, for two uncertainty levels, the fuzzy frequency response function for a displacement dof in an articulated truss structure.

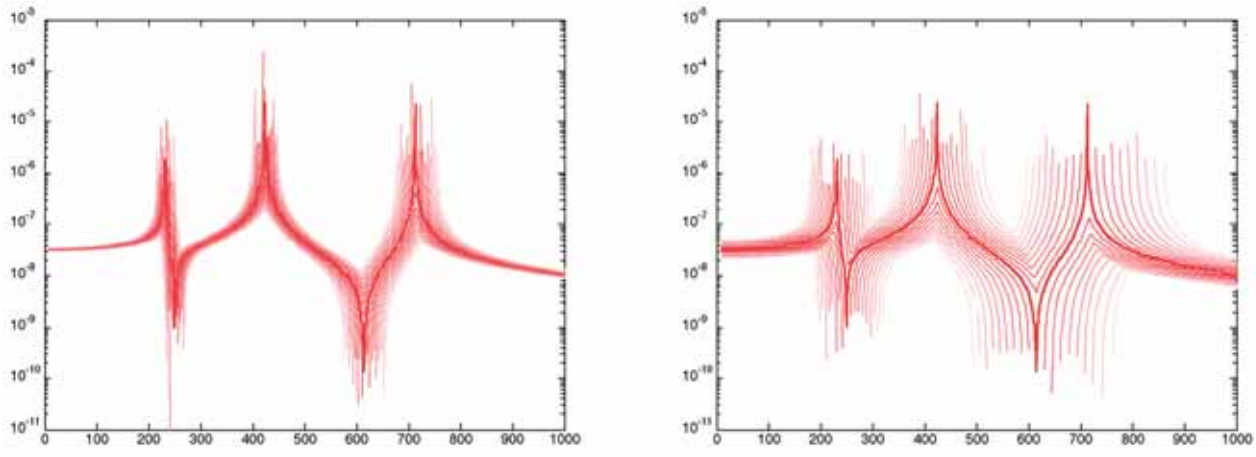

Fig. 3 : Articulated truss structure : fuzzy FRF of the displacement field (all parameters having 3% (left) or 10% (right) of variability)

Statistical Energy Analysis

FEM (or BEM) techniques are usually effective for 'low' frequency evaluations. This is mainly due to the need for mesh refinements as frequency increases. The related model size (number of displacement and pressure degrees of freedom) induced practical limitations that are, more and more, overcome by advanced solvers exploiting parallel architectures and powerful processors. This recent trends allows for a significant extension of frequency ranges where FEM techniques can be applied. Such extension calls however for appropriate post-processing techniques: local indicators are not meaningful in such a context since they are strongly spatially variable and too sensitive to model parameters. Global indicators (spatially and/or frequency averaged) are requested and can effectively be produced starting from extended modal representations of complex vibro-acoustic systems [17]. Such an approach is also the basis for developing automatic partitioning techniques [18,19,20] supporting the application of SEA techniques. Additionally such advanced FE models can support the evaluation of coupling loss factors and a more precise evaluation of the injected power related to complex random excitations (turbulent boundary layer for instance). An example of energetic post-processing on a train structure is provided in Fig. 4.

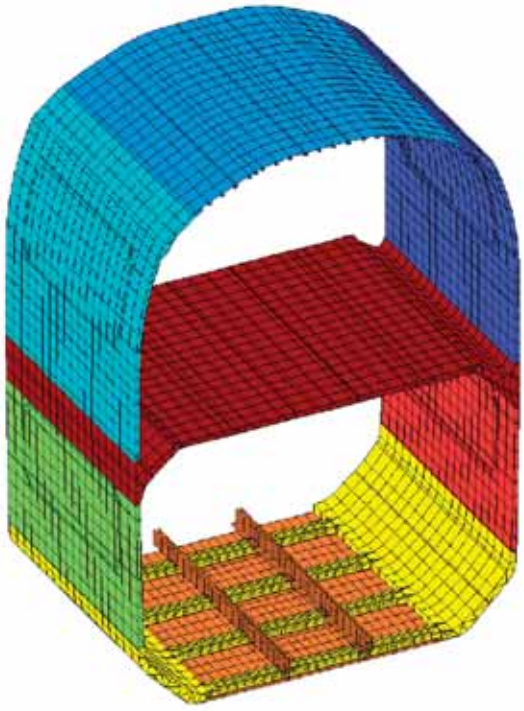

Fig 4 : Optimal SEA subsystems for a train structure model

Conclusions

Vibro-acoustic simulations are generally affected by several uncertainty sources appearing at different stages of the design process. In the early modeling stage, a continuous model is set up in simplification of a real physical behavior. The parameters of this model are, in most circumstances, not known with certainty. In a vibro-acoustic context, the response variability mainly results from a significant uncertainty in the structure characterization (uncertain geometric configuration) and in the excitation model (diffuse field and TBL). If sufficient information is available, a probabilistic behavior for these unknown parameters can be used (random variables or random processes) and appropriate solution technique can be used. For low-frequency analysis, the stochastic FEM provides efficient modal strategies that enable the computation of the first and second-order response statistics. Alternatively, fuzzy techniques can handle design variables for which the probabilistic model is not suitable. As the modal density increases, the response uncertainty is such that the vibro-acoustic response should be investigated in terms of spatial and/or frequency-averaged quantities. The resort to SEA techniques can be secured by post-processing modal FE-based results in order to automatically build an SEA model.

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